University of New Brunswick
Faculty of Computer Science

CS1303 Discrete Structures - Final Exam

April 17th, 2021;
Time Allowed: 180 minutes

Student Name: ____________________________ **Student No.:** _____________________

Instructions
This paper contains 6 questions, and comprises 4 pages.
Answer ALL questions. This is an open-book examination.
The marking scheme is shown in the left margin and [100] constitutes full marks.

The following table may be needed for taking this examination.

Given any statement variables p, q, and r, a tautology $\mathbf{t}$ and a contradiction $\mathbf{c}$, the following logical equivalences hold.

1.	*Commutative laws:*	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2.	*Associative laws:*	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3.	*Distributive laws:*	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4.	*Identity laws:*	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
5.	*Negation laws:*	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
6.	*Double negative law:*	$\sim(\sim p) \equiv p$	
7.	*Idempotent laws:*	$p \wedge p \equiv p$	$p \vee p \equiv p$
8.	*Universal bound laws:*	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
9.	*De Morgan's laws:*	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
10.	*Absorption laws:*	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11.	*Negations of* $\mathbf{t}$ *and* $\mathbf{c}$:	$\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$

In addition, Representation of If-Then as Or (Implication Equivalence) means
$p \rightarrow q \equiv \neg p \vee q$, which may also be used.

[**48**] 1. **Multiple choice questions:** read each question carefully and choose the correct answer: A, B, C or D. Make sure you only choose one answer for each question.

[4] (1) Find which of the following sentences is NOT a proposition: ________.

A. I passed the CS1303.

B. I did not pass the CS1303.

C. CS1303 is not a simple course.

D. CS1303 is a simple course, isn't it?

[4] (2) Let $A = \{a, b, c, d, e\}$. If we list all the strings of length 5 over A with at least four characters that are the same into a string set B. What is the size of set B ________.

A. 105 B. 125 C. 95 D. 115

[4] (3) Which of the following statements is false? ________

A. $x \in \{x, y, z\}$ B. $\{z\} \in \{x, y, \{z\}\}$

C. $\varnothing \subseteq \{\varnothing, x, y, \{z\}\}$ D. $\{x\} \subseteq \{\{x, y\}, z\}$

[4] (4) Let $\oplus$ denote as the *exclusive or* operation. Which of the following is not logically equivalent to $p \oplus q$? ________.

A. $(p \vee q) \wedge \neg(p \wedge q)$ B. $(p \wedge \neg q) \vee (\neg p \wedge q)$

C. $\neg((p \wedge q) \vee (\neg p \wedge \neg q))$ D. $(p \vee q) \vee (\neg p \wedge \neg q)$

[4] (5) Let $\oplus$ denote as the *exclusive or* operation. Which of the following compound propositions is a contradiction? ________.

A. $(\neg p \oplus \neg q) \wedge p$ B. $(p \oplus p) \wedge q$ C. $(q \oplus \neg q) \vee p$ D. $(p \oplus \neg p) \vee q$

[4] (6) Let $P(x)$ be the statement "$x^2 \geq x$". What is the truth value of the quantification $\forall x\ P(x)$ where the domain consists of all positive integers? What is the truth value of the quantification $\forall x\ P(x)$ where the domain consists of all positive real numbers? ________.

A. true, true B. true, false

C. false, true D. false, false

[4] (7) Let x and y be real numbers. What are the truth values of the following two propositions ________.

- $\forall x, \exists y, x > y$
- $\forall x, x \neq 0 \rightarrow \exists y, xy = 1$

A. true, true B. true, false

C. false, true D. false, false

[4] (8) Let x and y be real numbers. What are the truth values of the following two propositions ________.

- $\exists x, \forall y, xy = y$
- $\exists x, \forall y, x + y = 1$

A. true, true B. true, false

C. false, true D. false, false

[4] (9) What is the negation of the statement "There is an honest politician" ________.

A. All politicians are honest.
B. All politicians are not honest.
C. Some politicians are honest.
D. Some politicians are not honest.

[4] (10) For every integer $n \geq 3$, which of the following is equal to $\sum_{i=3}^{n} i(i!)$ ________?

A. $(n+1)!$ B. $(n+1)! - 1$ C. $(n+1)! - 2$ D. $(n+1)! - 6$

[4] (11) For any set S, the power set of S is denoted as $\mathcal{P}(A)$. Now, for some sets A, B, and C, the following is known: $|\mathcal{P}(A)| = 8$, $|\mathcal{P}(B)| = 16$, $|\mathcal{P}(C)| = 32$. How many elements are contained in the set $A \times B \times C$ ________?

A. 56 B. 4096 C. 12 D. 60

[4] (12) For finite sets A, B, C. It is known that $|A| = |B| = 16$, $|C| = 25$, $|A \cap B| = 10$, $|A \cap C| = |B \cap C| = 9$, $|A \cap B \cap C| = 3$. What is the value of $|A \cup B \cup C|$ ________?

A. 57 B. 88 C. 32 D. 82

[**10**] 2. Show that

$$(p \to t) \wedge (q \to t) \equiv (p \vee q) \to t$$

by

[5] (a) Using truth tables;

[5] (b) Using logical equivalences, and state clearly which law(s) you are using in each step.

[**10**] 3. The logician Raymond Smullyan describes an island containing two types of people: knights who always tell the truth and knaves who always lie. Now, you are visiting the island and have the following encounters with natives.

[5] (a) Two natives A and B approach you but only B speaks.

B says: <u>Both of us are knaves.</u>

What are A and B?

[5] (b) You then encounter natives C and D.

C says: <u>D is a knave.</u>

D says: <u>C is a knave.</u>

How many knaves are there?

[**15**] 4. Please prove the following statements.

[5] (a) If a square of some integer n is divisible by 7, then the integer n itself is divisible by 7.

[5] (b) $\sqrt{7}$ is an irrational number.

[5] (c) $\log_2 7$ is an irrational number.

[**10**] 5. Please use the proper mathematical induction (either the regular mathematical induction or the strong mathematical induction) to prove the following statements.

[5] (a) For every integer $n \geq 1$,

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n^2+n}{2}\right)^2.$$

[5] (b) Suppose $f_0, f_1, f_2, \cdots$ is a sequence defined as follows:

$$f_0 = 5, \quad f_1 = 16, \qquad f_k = 5f_{k-1} - 6f_{k-2} \qquad \text{for each integer } k \geq 2$$

Prove that $f_n = 6 \cdot 3^n - 2^n$ for every integer $n \geq 0$.

[**7**] 6. Find a counterexample to show that the each statement is false. Assume all sets are subsets of a universal set U.

[3] (a) For all sets A and B, $(A \cup B)^c = A^c \cup B^c$.

[4] (b) For all sets A, B, and C, if $B \cup C \subseteq A$ then $(A - B) \cap (A - C) = \varnothing$.

END OF PAPER